https://www.linkedin.com/feed/update/urn:li:activity:6519042498638282752

5519: **Proposed by Titu Zvonaru**, **Comănesti**, **Romania**

Let a,b,c be positive real numbers. Prove that

$$\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}+\frac{2abc}{a^3+b^3+c^3}\geq\frac{11}{3}$$

Solution by Arkady Alt, **San Jose**,**California**, **USA**.

Since by Cauchy inequality $\sum_{cyc}\frac{a^2}{b^2}=\sum_{cyc}\frac{a^4}{a^2b^2}\geq\frac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2}$

remains to prove inequality

(**1**) $$\frac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2}+\frac{2abc}{a^3+b^3+c^3}\geq\frac{11}{3}.$$

Let $p:=ab+bc+ca,q:=abc$. Then,assuming $a+b+c=1$ (due homogeneity of (**1**))

we obtain $\frac{(a^2+b^2+c^2)^2}{a^2b^2+b^2c^2+c^2a^2}+\frac{2abc}{a^3+b^3+c^3}-\frac{11}{3}=\frac{(1-2p)^2}{p^2-2q}+\frac{2q}{1+3q-3p}-\frac{11}{3}.$

Since $3p=3(ab+bc+ca)\leq(a+b+c)^2=1$, $9q\geq 4p-1$ (normalized by $a+b+c=1$ Schure's Inequality $\sum_{cyc}a(a-b)(a-c)\geq 0$ in p,q notation) and $\frac{(1-2p)^2}{p^2-2q}$, $\frac{2q}{1+3q-3p}$

both increasing in q ($\frac{3q}{1+3q-3p}=1-\frac{1-3p}{1+3q-3p}$) then

$$\frac{(1-2p)^2}{p^2-2q}+\frac{2q}{1+3q-3p}-\frac{11}{3}\geq\frac{(1-2p)^2}{p^2-2\cdot\frac{4p-1}{9}}+\frac{2\cdot\frac{4p-1}{9}}{1+3\cdot\frac{4p-1}{9}-3p}-\frac{11}{3}=$$

$$\frac{(p+2)(1-3p)^2}{(2-5p)(9p^2-8p+2)}\geq 0.$$